Optimal Pricing Strategy for Telecom Operator in Cellular Networks with Random Topologies

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***Abstract*—Traditional pricing of a cellular network operator considers the bandwidth utilization for users and operational cost. In this paper, we re-examine this critical subject through a holistic engineering view taken energy efficiency into account. We treat this issue as a two-stage Stackelberg game between the operator and mobile users, where the operator determines the price of unit bandwidth, and accordingly the users choose the amount of requested bandwidth to maximize their payoff. To reach a general result, randomly deployed base stations in large scale have been taken into consideration. By stochastic geometry, we derive the operator's profit and the energy efficiency per unit area, while considering the actual bandwidth demand of users based on their acceptable price. Simulation results show that the operators profit and the energy efficiency can be achieved their maximum at nearly the same price (i.e., optimal price), and the optimal pricing strategy varies with the deployment density of base stations.**

***Index Terms*—Optimal pricing, cellular network, energy efficiency, Stackelbery game.**

I. INTRODUCTION

The rapid increase of traffic motivates cellular operators to deploy dense networks. Traditionally, for the ease of mathematical modeling, the network structure has been assumed as a highly idealized grid-based model, where base stations (BSs) are located following a regular hexagonal lattice topology and the cell sizes of all BSs are the same. For idealized single-tier homogenous networks, the profit and energy efficiency have been maximized by optimizing traffic loads on BSs while taking into the user blocking probability [1]. In practical wireless networks, it has been found that the network topology is generally irregular and stochastic geometry tools have been used to model the spatial distributions of network entities [2]. While most exiting work mainly focused on the network performance in terms of coverage, spectral and energy efficiency [3], [4], the profit compared against the energy efficiency has not been sufficiently studied.

The economic aspects of the communication networks and the associated traffic offloading have been recently studied. In [5], the authors have investigated the optimal differentiated contracts in overlay macrocell-femtocell systems under both split-spectrum and shared-spectrum models. In [6], an incentive framework has been proposed in spectrum sharing femtocell networks, where the macro base station motivates femtocells to adopt hybrid access by pricing the accessed macrocell user equipments' transmission rates guaranteed by femtocell access points. In [7], pricing and spectrum allocation decisions for both macrocell and femtocell services have been made by operator to maximize the operator's total profit and user's payoff. Pricing strategies for operators in cognitive femtocell networks were discussed in [8]. The economics of cellular traffic offloading through third-party WiFi access points were modeled and analyzed in [9]. In [10], the authors investigated the economic effects of device-to-device (D2D) assisted offloading in cellular networks in which the operator's profit can be maximized in both the underlay and the overlay modes of D2D communications. However, the existing works concentrated on the operator's profit and the energy efficiency seperately, the combination of the both is limited in analysis given the density of BSs.

In this paper, we obtain the optimal pricing strategy per unit bandwidth for a cellular network operator. Since the users select the suitable frequency resources under the price per unit bandwidth announced by the operator, the interaction between them can be modeled as a sequential game. We derive the operator's profit and the energy efficiency per unit area by means of stochastic geometry, whilst maximizing user's payoff and taking the actual bandwidth demand of users based on their acceptable price into consideration. In addition, the maximum profit and energy efficiency with respect to the price is examined. Simulation results show that the operators profit and the energy efficiency can be achieved their maximum at nearly the same price (i.e., optimal price), and the optimal pricing strategy varies with the deployment density of base stations.

II. SYSTEM MODEL

We consider the cellular downlink network, where the macro BSs are distributed following a spatial Poisson point process (PPP) on the plane $\mathbb{R}^2$ with density λ_M [2], and the set of BSs is donated as $\Psi_M = \{b_j, j = 1, 2, ...\}$. Cellular users are located on $\mathbb{R}^2$ according to an independent homogenous PPP Ψ_U with intensity λ_u. We assume that the i^{th} mobile user is associated with its closest BS b_j and is denoted $u_{i,j}$. The coverage area of BS b_j can be defined as the set of $V_j = \left\{x \in \mathbb{R}^2 \,|\|x - b_j\| \le \|x - b_k\|, b_k \in \Psi_M \backslash b_j \right\}$, where $\|a - b\|$ represents the Euclidean distance between a and b , and b_j is assumed as the location of BS b_j. Therefore, the set of BSs Ψ_M forms the Voronoi tessellation.

Since the orthogonal frequency division multiple access is adopted in the downlink within a cell , there is no intra-cell interference. The number of cellular users served by BS b_j can be expressed as N_j. The system bandwidth (i.e., B Hz) is reused by all cells in the network, i.e., each cell has access to the B Hz bandwidth. We suppose the scheduler of BSs allocates bandwidth resources utilizing Round-Robin method according to users' demands. Moreover, we consider that every macro BS has a maximum transmission power denoted by P_M, and the transmit power is uniformly distributed among the sub-carriers. We assume that the user can obtain a fixed power P_M/B for each occupied bandwidth. When the bandwidth is fully utilized by users based on their current demands, the BSs will transmit at power P_M, otherwise the aggregate transmission power at BSs is less than the maximum transmission power P_M.

The channel model consists of small-scale fast fading and path loss exponent α , i.e., $h_{i,j}{r_{i,j}}^{-\alpha}$, where the channel power gain $h_{i,j}$ follows an exponential distribution with unit mean, i.e., $h_{i,j} \sim \exp(1)$, $r_{i,j} = \|u_{i,j} - b_j\|$ denotes the distance link between $u_{i,j}$ and b_j , and α is the path loss exponent.), the effects of thermal noise is neglected. Not necessary, the signal-to-interference ratio of $u_{i,j}$ can be expressed as

$$SIR_{u_{i,j}} = \frac{h_{i,j}{r_{i,j}}^{-\alpha}}{\sum_{k\in\{\Psi_M \setminus b_j\}} h_{i,k}{r_{i,k}}^{-\alpha}}, \quad (1)$$

where we eliminate the power of serving BS and interfering BSs in above fraction by considering their values are the same according to the uniformly power allocation, and the denominator is the cumulative interference from all other BSs transmitting in the same resource block as used by $u_{i,j}$.

Moreover, the spectral efficiency (SE) of user $u_{i,j}$ is given by

$$\theta_{u_{i,j}} = \log_2(1 + SIR_{u_{i,j}}). \quad (2)$$

By obtaining b Hz spectrum, user $u_{i,j}$ can achieve the data rate $b\theta_{u_{i,j}}$ bits per second. We define the utility of user $u_{i,j}$ as $\ln(1 + \theta_{u_{i,j}} D(b, \tau))$, where $D(b, \tau) = b(\tau/\tau_0)^{-\eta}$ is the demand function, b denotes the user's original desired bandwidth without considering the price, τ indicates the price per unit bandwidth determined by the operator, τ_0 represents user's psychology price, and η is the demand elasticity ($\eta > 1$ and η is constant) [12]. Such a logarithmic user $u_{i,j}$ function has been widely used in the literature to depict the diminishing returns of getting more resources. Thus, the user's payoff can be expressed as follows:

$$\pi_{user} = \ln(1 + \theta D(b, \tau))\xi_R - D(b, \tau)\tau, \quad (3)$$

where ξ_R is the income factor that reflects the relationship between the utility and money. More specifically, a higher value of ξ_R indicates users prefer to purchase more traffic data to use. $D(b, \tau)\tau$ is the linear payment for a cellular user.

The operator's profit function on a unit area is given by

$$\pi_{operator} = \lambda_u^{act} b_{allocation}\tau - \lambda_M P_{total}\varphi - F, \quad (4)$$

Table I: List of Main System Parameters Notations

Notation	Defination
Ψ_M, Ψ_U	Sets of BSs and users
λ_M, λ_u	Densities of BSs and users
V_i	Coverage area of BS b_j
N_j	The number of users served by BS b_j
B	Total bandwidth of a BS
P_M	Maximum BS transition power
$\theta_{u_{i,j}}$	User $u_{i,j}$'s spectral efficiency
τ	The price per unit bandwidth
τ_0	User's psychology price
b	User's original desired bandwidth
$D(b, \tau)$	User's radio resource demand function
π_{user}	User's payoff
ξ_R	User's income factor
$\pi_{operator}$	Operator's profit
λ_u^{act}	Active cellular users' density
b_{max}	Maximum resources allocated to a typical user
P_{max}	Total power consumption at a BS
P_{OM}	Non-transmission power
P_M^{agg}	Aggregate transmission power
$\mathbb{P}_M^{act}$	Active probability of BSs
φ	Cost factor of the power consumption at BSs

where λ_u^{act} denotes the intensity of active mobile users that are willing to purchase bandwidth according to the user's payoff in (3); $b_{allocation} = \min(D(b, \tau), b_{max})$, where b_{max} denotes the maximum resources allocated to a typical user which will be discussed latter; P_{total} indicates the total power consumption at a BS; which includes the non-transmission power P_{OM} and the aggregate transmission power P_M^{agg} (i.e., $P_{total} = P_{OM} + \mathbb{P}_M^{act} P_M^{agg}$, where $\mathbb{P}_M^{act} = 1 - [1 + \lambda_u/(K\lambda_M)]^{-K}$ is the average probability of a cell having at least one user to serve with $K = 3.575$ [13]); φ denotes a cost factor of the power consumption, and F represents other fixed operating cost, such as rental cost.

Table I summarizes the main notations used in this paper.

III. PROBLEM FORMULATION AND SOLUTION

Because of the relationship between operator and users is similar to that between the leader and the followers in the two-stage game theory, in this section, we will look at the two-stage Stackelberg game. In stage I, the operator announces the price of unit bandwidth to cellular users. In stage II, each user decides how much bandwidth to request. We will analyze this game by using backward induction.

A. Cellular user's bandwidth in stage II

In order to obtain the optimal bandwidth that can maximize a user's payoff, we take the first derivative of π_{user} with respect to the original desired bandwidth b as follows:

$$\frac{\partial \pi_{user}}{\partial b} = \frac{\theta\tau^{-\eta}\xi_R}{{\tau_0}^{-\eta} + \theta b\tau^{-\eta}} - \frac{\tau^{1-\eta}}{{\tau_0}^{-\eta}}. \quad (5)$$

Then we let $\frac{\partial \pi_{user}}{\partial b} = 0$, solve it for b, and obtain

$$b^* = \frac{(\theta\xi_R - \tau){\tau_0}^{-\eta}}{\theta\tau^{1-\eta}}. \quad (6)$$

In (6), the optimal bandwidth b^* is increasing the ξ_R and decreasing of τ_0. Recall that ξ_R also reflects the data demand of cellular users, i.e., a user urgently needs the data traffic that results in requesting more bandwidth to satisfy its demand. Note that, according to (6), user $u_{i,j}$ whose spectral efficiency $\theta_{u_{i,j}} > \frac{\tau}{\xi_R}$ will acquire some bandwidth in stage II; otherwise it will give up the current connection until it moves to a place where its $\theta_{u_{i,j}}$ goes beyond the above threshold.

Plugging (6) in the user's demand function, we obtain the actual bandwidth requirement at a user

$$D(b^*,\tau) = b^*(\frac{\tau}{\tau_0})^{-\eta} = \frac{\theta\xi_R - \tau}{\theta\tau}. \tag{7}$$

In (7), the user's optimal requirement is independent of the demand elasticity η. Since the user's payoff π_{user} is a concave function of $D(b,\tau)$ (i.e., $\frac{\partial^2 \pi_{user}}{\partial D(b,\tau)^2} < 0$), if the serving BS cannot satisfy the user's optimal requirement (i.e., $D(b^*,\tau) > b_{max}$), then the user would prefer to be allocated with b_{max} rather than interrupt the connection.

B. Operator's pricing in stage I

In stage I, aiming at maximizing the profit of the operator, we will analyze the operator's average income as well as cost on unit area as functions of the price and other network parameters.

Note that θ is a random variable in a user's optimal requirement $D(b^*,\tau)$. The cumulative distribution function (CDF) of θ (i.e., $\Pr[\theta > t\,|x]$) is used to derive the conditional probability density function (CPDF), which is conditioned on the serving link distance x, as follows:

$$\begin{aligned}\Pr[\theta > t\,|x] &= \Pr\left[h_{i,j} > x^\alpha I_r\left(2^t - 1\right)\right]\\ &\overset{(a)}{=} \mathcal{L}_{I_r}\left(x^\alpha\left(2^t-1\right)\right)\\ &\overset{(b)}{=} \exp\left\{-\pi\lambda_M Z(t,\alpha)\,x^2\right\},\end{aligned} \tag{8}$$

where $Z(t,\alpha) = \left(2^t-1\right)^{\frac{2}{\alpha}} \int_{(2^t-1)^{-2/\alpha}}^{\infty} \frac{1}{1+u^{\alpha/2}}du$, I_r indicates the aggregate interference generated by interfering BSs at $u_{i,j}$, (a) is obtained using properties of exponential distribution $h_{i,j} \sim \exp(1)$, and (b) is obtained based on [2] with SINR in [2] replaced by θ in this paper. Then, we obtain the CPDF of θ as

$$\begin{aligned}&f_\theta(t\,|x) = \frac{\partial F_\theta(t\,|x)}{\partial t}\\ &= \frac{\partial}{\partial t}\left[1 - \Pr(\theta > t\,|x)\right]\\ &= \frac{-\partial}{\partial t}\exp\left(-\pi\lambda_M x^2\left(2^t-1\right)^{\frac{2}{\alpha}}\int_{(2^t-1)^{-2/\alpha}}^{\infty}\frac{1}{1+u^{\alpha/2}}du\right)\\ &= \pi\lambda_M x^2 \ln 2 \frac{2}{\alpha}\left(\frac{2^t}{2^t-1}Z(t,\alpha)+1\right)e^{-\pi\lambda_M x^2 Z(t,\alpha)}.\end{aligned} \tag{9}$$

Since the locations of BSs and users following independent PPPs, the null probability is $\mathbb{P}[x\,|X] = e^{-\pi\lambda_M X^2}$. Therefore, the probability density function (PDF) of the serving link distance is given by

$$\begin{aligned}f(x) &= \frac{\partial F(x)}{\partial x} = \frac{\partial\left(1-\mathbb{P}[x\,|X]\right)}{\partial x}\\ &= \frac{\partial\left(1-e^{-\pi\lambda_M x^2}\right)}{\partial x}\\ &= 2\pi\lambda_M x e^{-\pi\lambda_M x^2}.\end{aligned} \tag{10}$$

Recall that a cellular user connects to the nearest BS as long as its spectral efficiency is higher than a threshold which is determined by the price τ and ξ_R (i.e., $\theta > \frac{\tau}{\xi_R}$). Combining (7), (9) and (10), we obtain the expectation of the actual required bandwidth $D(b^*,\tau)$ as follows:

$$\begin{aligned}&\mathbb{E}\left[D\left(b^*,\tau\right)\right] = \int_{\frac{\tau}{\xi_R}}^{\theta_{\max}}\int_{x_{\min}}^{\infty} D\left(b^*,\tau\right) f_\theta(t\,|x) f(x)\,dxdt\\ &= \int_{\frac{\tau}{\xi_R}}^{\theta_{\max}} \frac{t\xi_R - \tau}{t\tau}\left(\frac{2^t}{2^t-1}Z(t,\alpha)+1\right)\\ &\quad\cdot \ln 2 \frac{4}{\alpha}\left(\pi\lambda_M\right)^2 \int_{x_{\min}}^{\infty} x^3 e^{-\pi\lambda_M x^2[Z(t,\alpha)+1]}dxdt\\ &= \frac{2\ln 2}{\tau\alpha}\int_{\frac{\tau}{\xi_R}}^{\theta_{\max}}\frac{t\xi_R-\tau}{t}\left(\frac{2^t}{2^t-1}Z(t,\alpha)+1\right)\\ &\quad\cdot\frac{\Gamma\left(2,\pi\lambda_M\left[Z(t,\alpha)+1\right]x_{\min}^2\right)}{\left[Z(t,\alpha)+1\right]^2}dt,\end{aligned} \tag{11}$$

where $\Gamma(z,x)$ is the incomplete gamma function (i.e., $\Gamma(z,x) = \int_x^\infty t^{z-1}e^{-t}dt$), x_{min} is the minimum distance between a cellular user and its associated BS, and θ_{max} is the spectral efficiency corresponding to x_{min}, i.e.,

$$\begin{aligned}&\theta_{\max} = \mathbb{E}_{h_{i,j}}\left[\log\left(1+\frac{h_{i,j}(x_{\min})^{-\alpha}}{\sum\limits_{k\in\Psi_M\backslash b_j} g_{i,k}x_{i,k}^{-\alpha}}\right)\right]\\ &\overset{(a)}{=}\int_0^\infty \mathbb{P}\left[\log_2\left(1+\frac{h_{i,j}(x_{\min})^{-\alpha}}{\sum\limits_{k\in\Psi_M\backslash b_j} g_{i,k}x_{i,k}^{-\alpha}}\right) > t\right]dt\\ &\overset{(b)}{=}\int_0^\infty \mathbb{P}\left[h_{i,j} > (x_{\min})^\alpha I_r\left(2^t-1\right)\right]dt\\ &\overset{(c)}{=}\int_0^\infty \mathcal{L}_{I_r}\left[\frac{(x_{\min})^\alpha}{2^t-1}\right]dt\\ &= \int_0^\infty \frac{dt}{e^{\pi\lambda_M Z(t,\alpha)(x_{\min})^2}},\end{aligned} \tag{12}$$

where (a) follows from $\mathbb{E}[X] = \int_0^\infty \mathbb{P}[X > t]dt$ for $X > 0$, (b) and (c) are obtained by substituting the minimum distance x_{min} in (8).

Considering the limited resources of a cellular network, the total required bandwidth should not exceed the system bandwidth. The average number of active mobile users, that meet $\theta > \frac{\tau}{\xi_R}$, within a cell area V_j of BS b_j is given by

$$\begin{aligned}\overline{N} &= \frac{\lambda_u \Pr\left[\theta > \frac{\tau}{\xi_R}\right]}{\lambda_M} \\ &= \frac{\lambda_u}{\lambda_M}\int_{x_{\min}}^{\infty} \Pr\left[\theta > \frac{\tau}{\xi_R}\middle| x\right] f(x)\, dx \\ &= \int_{x_{\min}}^{\theta} \frac{2\pi\lambda_u x dx}{e^{\pi\lambda_M\left[Z\left(\frac{\tau}{\xi_R},\alpha\right)+1\right]x^2}} \\ &= \frac{\lambda_u e^{-\pi\lambda_M \Omega x_{\min}{}^2}}{\lambda_M \Omega},\end{aligned} \quad (13)$$

where $\Omega = Z\left(\frac{\tau}{\xi_R},\alpha\right)+1$. Therefore, the maximum resources that can be allocated to a typical user, i.e., $b_{\max}$ in (4) , is given by $b_{\max} = B/\overline{N}$, and the intensity of active users is $\lambda_u^{act} = \overline{N}\lambda_M$. The average income of the operator per unit area is given by

$$\mathcal{O}_{income} = \lambda_u^{act} \cdot \min\left(\mathbb{E}\left[D\left(b^*,\tau\right)\right], b_{\max}\right) \cdot \tau. \quad (14)$$

Substituting (11), (12) and (13) in (14), we obtain the operator's income, which can be evaluated numerically.

Next, we characterize the average aggregate transmission power at a BS influenced by price. Once the total users' bandwidth demand is more than supply, BS will transmit at maximum power P_M. This can be represented by the following expression

$$\mathbb{E}\left[P_M^{agg}\right] = \min\left(\mathbb{E}\left[D\left(b^*,\tau\right)\right] \cdot \overline{N}, B\right) \cdot \frac{P_M}{B}. \quad (15)$$

The telecom operator's cost per unit area is given by

$$\mathcal{O}_{\cos t} = \lambda_M P_{OM}\varphi + \lambda_M \mathbb{P}_M^{act}\mathbb{E}\left[P_M^{agg}\right]\varphi + F. \quad (16)$$

In (16), the first term denotes the cost of non-transmission power consumption, the second term denotes the cost of aggregate transmission power consumption at active BSs, and the last term represents other fixed cost. Finally, the operator's profit is formulated as

$$\pi_{operator} = \mathcal{O}_{income} - \mathcal{O}_{\cos t}. \quad (17)$$

C. Analysis of the energy efficiency

Energy efficiency has also become an important performance metric of cellular networks due to the significant increase in energy consumption for meeting traffic demands. After analyzing the operator's profit and user's utility, the energy efficiency is analyzed from the perspective of network bandwidth's pricing strategy. According to Shannon's theorem, the data rate of a typical user can be represented as $R_{u_{i,j}} = b_{allocation}\log_2\left(1+SNR_{u_{i,j}}\right)$. Thus we get the expectation of the data rate as follows:

$$\begin{aligned}\mathbb{E}\left[R_{u_{i,j}}\right] &= \mathbb{E}\left[b_{allocation}\log_2\left(1+SNR_{u_{i,j}}\right)\right] \\ &= \min\left(\mathbb{E}\left[D\left(b^*,\tau\right)\right], b_{\max}\right) \cdot \mathbb{E}\left[\theta_{u_{i,j}}\right],\end{aligned} \quad (18)$$

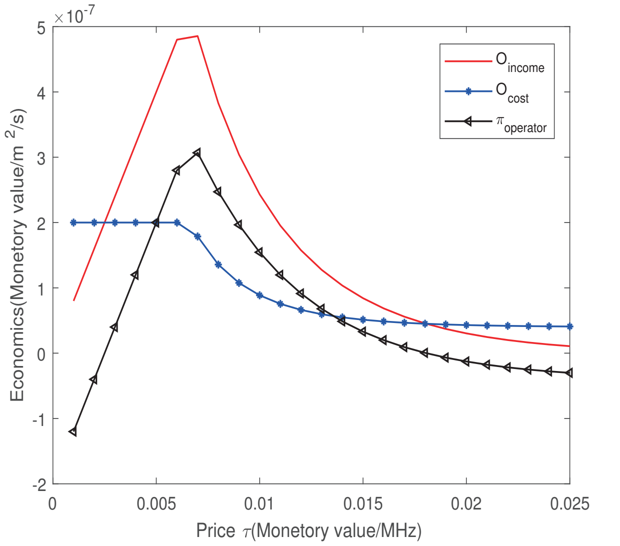

Fig. 1. The operator's income/cost/profit per unit area versus the price τ for $\alpha = 4$.

where the average value of spectral efficiency $\mathbb{E}\left[\theta_{u_{i,j}}\right]$ is given by [2] with some modifications.

$$\begin{aligned}&\mathbb{E}\left[\theta_{u_{i,j}}\right] = \mathbb{E}\left[\log_2\left(1+SNR_{u_{i,j}}\right)\right] \\ &= \int_0^{\infty} e^{-\pi\lambda_M x^2}\int_0^{\infty} \mathcal{L}_{I_r}\left(x^{\alpha}\left(2^t-1\right)\right) dt 2\pi\lambda_M x dx \\ &= \int_0^{\infty}\int_0^{\infty} 2\pi\lambda_M x e^{-\pi\lambda_M x^2[Z(t,\alpha)+1]} dx dt \\ &= \int_0^{\infty} \frac{dt}{1+\left[Z\left(t,\alpha\right)+1\right]},\end{aligned} \quad (19)$$

where $Z(t,\alpha)$ can be found in (8). We can obtain the network energy efficiency, and can be expressed as:

$$\eta_{EE} = \frac{\min\left(\mathbb{E}\left[D\left(b^*,\tau\right)\right], b_{\max}\right) \cdot \mathbb{E}\left[\theta_{u_{i,j}}\right] \cdot \lambda_u}{\lambda_M P_{OM} + \lambda_M \mathbb{P}_M^{act}\mathbb{E}\left[P_M^{agg}\right]}. \quad (20)$$

IV. SIMULATION RESULTS AND DISCUSSIONS

In this section, we present the numerical results to validate the theoretical analysis. We select $\lambda_u = 1 \times 10^{-3} users/m^2, \lambda_M = 1 \times 10^{-5} BSs/m^2, \alpha = 4, P_M = 40W, P_{OM} = 10W, x_{\min} = 20m, B = 8MHz, \xi_R = 5 \times 10^{-3}, \eta = 1.5, \varphi = 4 \times 10^{-4}$ unless specified otherwise.

Fig. 1 shows the economics of random topology network (i.e, income, cost and profit of telecom operator) with various price τ. Intuitively, there exits a price τ^* that can maximize the operator's profit. More specifically, if $\tau < \tau^*$, and when the user's actual bandwidth demand exceeds maximum allocated resources (i.e., $\mathbb{E}\left[D\left(b^*,\tau\right)\right] > b_{\max}$), the total bandwidth of BSs is allocated to users, thus operator's income is proportional to price. Because operator's cost remains the same, these results bring about the linear increase in profit. When $\mathbb{E}\left[D\left(b^*,\tau\right)\right] < b_{\max}$, user's bandwidth demand and

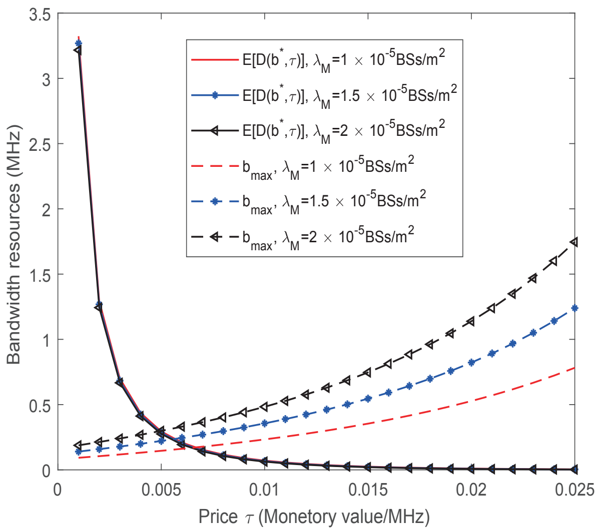

Fig. 2. The average actual required bandwidth and maximum allocated bandwidth per user on unit area versus the price τ for $\alpha = 4$.

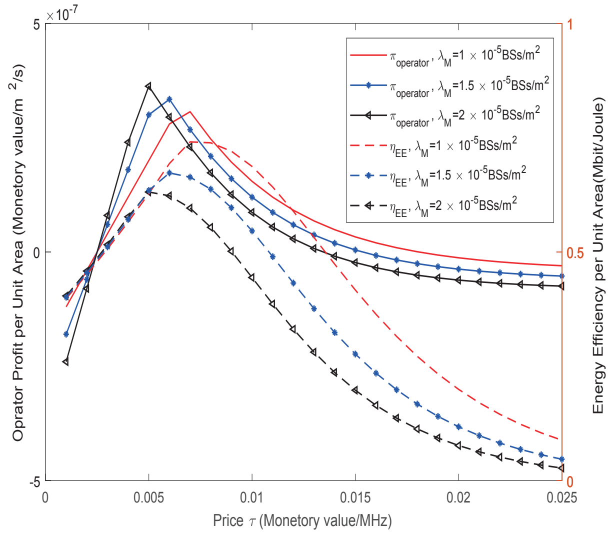

Fig. 3. The operator's profit and the network's energy efficiency per unit area versus the price τ for different intensities of BSs and $\alpha = 4$.

operator's cost decline rapidly while operator's profit increases slowly with respect to price τ. If $\tau > \tau^*$, operator's income dominates the profit due to the significant reduction of user's demand bandwidth, which leads to a decline in income with the increase of price τ. Finally, operator's cost will be close to a constant, that is, the cost of non-transmission power consumption, while income approaches zero.

Fig. 2 shows the user's actual required bandwidth $\mathbb{E}\left[D\left(b^*,\tau\right)\right]$ is mainly affected by the price, and the changes of BSs' density have little influence on it, it is consistent with the actual situation. By $b_{\max} = B\big/\overline{N} = \lambda_M \Omega e^{\pi\lambda_M\Omega {x_{\min}}^2}\big/\lambda_u$ and simulation figure of b_{max} can be seen that changes in the price and the BSs' density could make a significant change in maximum allocated bandwidth per user b_{max}, and b_{max} is an increasing function with respect to BSs' density.

Fig. 3 illustrates operator's profit and energy efficiency versus τ under different BSs intensities. Note that higher BSs intensity can achieve greater profit, and the upward trend in the profit growth phase is proportional to the corresponding BSs' density. Therefore, operator can increase the BSs' density to promote its profit. However, since the total power consumption increases with the BSs' density and there exists non-transmission power, the increase of data rate is slower than total power, so the maximum value of energy efficiency decreases as the BSs' density increases. The operator's profit and the energy efficiency achieve their maximum at nearly the same price, and in order to improve the operator's profit and the energy efficiency simultaneously, the density of BSs and the price should be appropriate. For example, operator can set the price between the one maximizes the operator's profit and the other maximizes the energy efficiency. In conjunction with Fig. 2 and Fig. 3, the order in which the operator's profit and the energy efficiency reach the maximum at three BS densities is consistent with the order in which the user's actual demand and maximum allocated resources intersect at three densities.

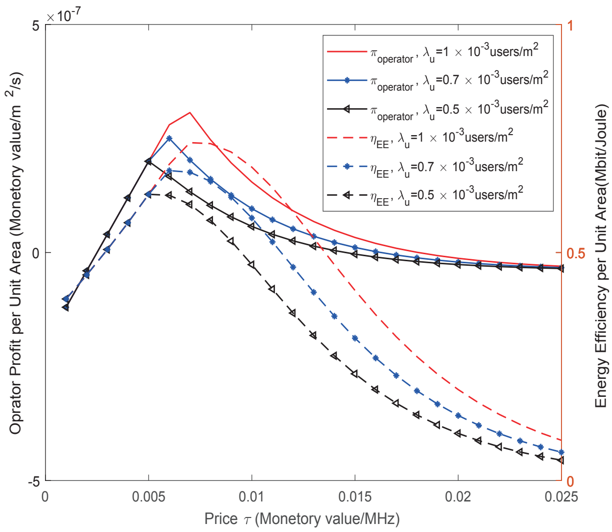

Fig. 4. The operator's profit and the network's energy efficiency per unit area versus the price τ for different intensities of users and $\alpha = 4$.

As illustrated in Fig. 4, when the price is low, the operator's profit and the energy efficiency are independent to price, since the bandwidth and power of BSs is fully allocated to its serving users, and the income and cost remain unchanged. In addition, the maximums of operator's profit and energy efficiency increase with the increase of users' density, thus the operator should try to attract more users to utilize its wireless network.

V. CONCLUSIONS

In this paper, we study a pricing scheme for telecom operator in large-scale cellular networks. By leveraging stochastic geometry and game theory, we derive the fluctuation trend of operator's profit and the energy efficiency with respect to the price per unit bandwidth. To improve both the operator's profit and energy efficiency, we obtain the optimal pricing strategy for a given BSs' density. Finally, we also validate the analytical results by numerical simulations and give the system-level pricing insights in Section IV.

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