Performance Analysis of Wireless-Powered Cellular Networks with Randomly Deployed Power Beacons

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***Abstract*—Wireless-powered cellular networks (WPCNs) emerge as a promising technology to satisfy the sufficiency of available energy at mobile devices. In WPCNs, a mobile device is charged from energy stations called power beacons (PBs) by microwave radiation, and it can harvest energy from ambient radio frequency (RF) of base stations etc., which suggests an energy-efficient way for communications. In this paper, we study an analytical model of the WPCNs with randomly deployed power beacons (PBs). The impacts of users' data rate requirements on both uplink and downlink transmissions are characterized, where uplink users capture energy from both PBs and BSs to maintain their transmit power. Considering the maximum allowable transmit power (MATP) of mobile device, we derive the successful transmission probability of an uplink cellular user who intends to harvest enough energy to transmit data meeting its quality of service (QoS) requirement. In addition, different modes of PBs are investigated in terms of the distributions of harvested energy, where a PB can either radiate energy isotropically or directionally towards users resorting to beamforming, called *isotropic* mode or *directed* mode, respectively. Numerical results validate our theoretical analysis and provide design insights to the WPCNs.**

***Index Terms*—Cellular networks, wireless power transfer, power beacons, outage probability.**

I. INTRODUCTION

Wireless charging emerges as one of the most attractive new features for mobile devices as it aims to compensate low battery state of devices by extracting energy from the ambient environment. Earlier efforts have been done in the PB-assisted WPCNs. The networks with a single dedicated power beacon as the energy source have been investigated in several works [1]-[3]. Different from the works aforementioned, [4] considered a single BS and a single PB co-existing in the network, and proposed an energy beamforming scheme in a MISO heterogeneous network. Taking into account the fact that, in general, there are multiple PBs existing in the WPCNs, [5]-[7] further studied the cellular-network architecture with randomly deployed PBs based on stochastic geometry.

In addition to harvesting energy from PBs, the cellular users can also collect energy from ambient RF radiated from BSs, which takes full advantage of the wireless energy in the environment and supplies an energy-efficient way for communications. However, the WPCNs integrated with both PBs and BSs as energy sources have not been well investigated. Furthermore, the performance of WPCNs with the impact of data rate requirement on system level like outage probability has not been well taken into consideration.

In this paper, we analyze the WPCNs of randomly deployed PBs by considering that uplink users can harvest energy from both PBs and BSs before their transmission. Incorporating the practical data rate requirement as the quality of service (QoS), we assume that users and BSs can be capable of performing adaptive power control to guarantee the required QoS. According to this mechanism, we characterize the distribution and expectation of the desired transmit power at a typical uplink user under the constraint of maximum allowable transmit power (MATP). In addition, the distributions of harvested energy at a typical uplink user are analyzed with different modes of PBs, i.e., isotropic mode and directed mode. Furthermore, the successful transmission probability for uplink user is obtained under a pre-determined QoS requirement.

Compared with the existing literatures, the main contributions are summarized as follows:

- We jointly analyze the randomly deployed PBs signals and ambient RF signals from BSs combined as the harvested energy in the large-scale networks, which is more approximate to the actual deployments of PBs and BSs.
- Jointly distributions of harvested energy under isotropic and directed modes of PBs are obtained, based on which we derive the exact expression of the successful transmission probability. It can provide an effective method for future research on other network's performance metrics, e.g., energy efficiency.
- The practical traffic demands of cellular users in the PB-assisted WPCNs are considered to satisfy their QoS requirements.

II. SYSTEM MODEL

We consider a WPCN with BSs and PBs randomly deployed as Fig. 1, where there are uplink and downlink users. BSs are modeled as a homogeneous Poisson Point Process (PPP) on the entire plane $\mathbb{R}^2$ with density λ_b, and that can be denoted as the set of $\Phi_B = \{b_j, j = 0, 1, 2, ...\}$. BS has MATP generally represented as P_b. Uplink and downlink users are spatially scattered on $\mathbb{R}^2$ according to independent homogeneous PPPs Φ_u^{UL} and Φ_u^{DL} with densities λ_u^{UL} and λ_u^{DL}, respectively. For each uplink user, mobile device has the MATP generally denoted by P_u. Furthermore, PBs are deployed following another PPP with the density of λ_p denoted by Φ_P. We assume each PB has a constant radiation power P_h, and radiates isotropically or directionally.

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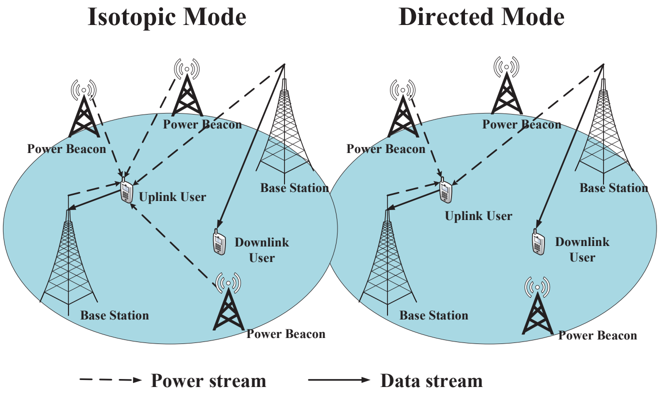

Fig. 1. System model

A. Cellular System

We assume each user connects to its nearest BS, and the cell area of j^{th} BS (b_j) can be defined as the set of $V_j = \left\{ x \in \mathbb{R}^2 \,|\|x - b_j\| \leq \|x - b_k\|, b_k \in \Phi_B \backslash b_j \right\}$, where $\|a - b\|$ represents the Euclidean distance between a and b. Uplink and downlink users in cell V_j can be denoted as the sets of $\Phi_{u,j}^{UL}$ and $\Phi_{u,j}^{DL}$, where $\left|\Phi_{u,j}^{UL}\right| = N_j^{UL}$ and $\left|\Phi_{u,j}^{DL}\right| = N_j^{DL}$ are the number of cellular users for uplink and downlink in cell V_j, respectively. We consider the Frequency Division Duplexing (FDD) mode for cellular system, where the bandwidths for uplink and downlink are B^{UL} and B^{DL} which can be equally allocated among mobile users (e.g., i^{th} uplink user in j^{th} cell obtains $B_{i,j}^{UL} = B^{UL}/N_j^{UL}$). We assume the PBs occupy different frequency bands from the cellular transmissions, so that the interferences among them can be eliminated. BSs and devices can be capable of performing adaptive power control. Therefore, according to Shannon's theorem, the desired transmit power $P_{i,j}^{UL}$ of i^{th} uplink user in j^{th} cell ($u_{i,j}^{UL}$) on its sub-band ($B_{i,j}^{UL}$) should ensure the required data rate $R_{i,j}^{UL}$ as follows:

$$R_{i,j}^{UL} = B_{i,j}^{UL}\log_2\left(1 + \frac{P_{i,j}^{UL}h_{i,j}^{UL}\left\|u_{i,j}^{UL} - b_j\right\|^{-\alpha}}{I_{i,j}^{UL} + \sigma^2}\right) \tag{1}$$

where $I_{i,j}^{UL} = \sum_{u_n^{UL} \in \Phi_{I_{i,j}}^{UL}} P_n^T g_n^{UL}\left(d_n^{UL}\right)^{-\alpha}$ is the interference, $h_{i,j}^{UL}$ and g_n^{UL} represent the fast fading coefficients under independent and identically distribution (i.i.d.) (i.e., $h_{i,j}^{UL}$, $g_n^{UL} \sim$ exp(1)), α denotes the path-loss exponent, $\Phi_{I_{i,j}}^{UL}$ indicates the set of interfering users for uplink, P_n^T expresses the transmit power at each interfering mobile device, $d_n^{UL} = \left\|u_n^{UL} - b_j\right\|$ represents the distance between the uplink interfering user u_n^{UL} and its receiver BS b_j, and σ^2 is positive and denotes the channel-noise variance.

For cellular downlink, we consider the case that the interferers transmit on their average power. In addition, the BS desired transmit power $P_{i,j}^{DL}$ for user $u_{i,j}^{DL}$ on its sub-band is allocated to guarantee the required data rate $R_{i,j}^{DL}$ as follows:

$$R_{i,j}^{DL} = B_{i,j}^{DL}\log_2\left(1 + \frac{P_{i,j}^{DL}h_{i,j}^{DL}\left\|u_{i,j}^{DL} - b_j\right\|^{-\alpha}}{I_{i,j}^{DL} + \sigma^2}\right) \tag{2}$$

where $I_{i,j}^{DL} = \sum_{b_k \in \Phi_B / b_j} \frac{P_b^{agg}}{N_j^{DL}} g_k^{DL}\left(d_k^{DL}\right)^{-\alpha}$, $h_{i,j}^{DL}$ and g_k^{DL} represent the fast fading factors following i.i.d. of exp(1), P_b^{agg} is the expectation of the aggregated transmit power of j^{th} BS b_j, denoted as P_j^{DL}, (i.e., $P_b^{agg} = \mathbb{E}\left[P_j^{DL}\right]$), and $d_k^{DL} = \left\|u_{i,j}^{DL} - b_k\right\|$ denotes the interfering distance between the downlink user $u_{i,j}^{DL}$ and k^{th} BS b_k.

B. Harvested Energy

We assume that uplink users transmit by using their instantaneous harvested energy. Consider the isotropic mode, where a typical user u_0 can harvest energy from all PBs and BSs. The instantaneous received power at u_0 is given by

$$\begin{aligned} P_H = P_H^{PB} + P_H^{BS} = \eta \sum_{p_j \in \Phi_P} P_h g_j^h \|p_j - u_0\|^{-\beta} \\ + \eta \sum_{b_j \in \Phi_B} P_j^{DL} g_j^{DL} \|b_j - u_0\|^{-\alpha} \end{aligned} \tag{3}$$

where η is the efficiency of the conversion from wireless power to circuit power, g_j^h and g_j^{DL} denotes the fast fading factors, $\|p_j - u_0\|$ represents the distance between j^{th} PB p_j and u_0, and β is the path-loss exponent for energy transmission from PBs to users. It is worth noting that the carrier frequencies of BSs downlink signals and PBs energy signals are different. This may indicate that the path-loss indexes of these two types of signals (i.e., α and β) should be clarified separately.

With directed mode of PBs, each mobile device can be charged by the nearest PB with beamforming. Therefore, the instantaneous received power at the typical user u_0 is

$$\begin{aligned} P_H = P_H^{PB} + P_H^{BS} = \eta G_m P_h g_j^h \|p_j - u_0\|^{-\beta} \\ + \eta \sum_{b_j \in \Phi_B} P_j^{DL} g_j^{DL} \|b_j - u_0\|^{-\alpha} \end{aligned} \tag{4}$$

where G_m represents the efficiency of the beamforming, and p_j denotes the nearest PB for the typical user u_0.

III. ANALYSIS OF HARVESTED ENERGY

In this section, we characterize the BS average aggregated transmit power and the distribution of the harvested energy P_H with isotropic mode and directed mode of PBs respectively.

A. Transmit Power of BS

The average desired transmit power at BS for user $u_{i,j}^{DL}$ can be derived by (2) conditioned on the service link distance as:

$$\mathbb{E}\left[P_{i,j}^{DL}\left|r_{i,j}^{DL}\right.\right] = \mathbb{E}\left[\frac{\left(2^{R_{i,j}^{DL}/B_{i,j}^{DL}} - 1\right)}{\left(I_{agg}^{DL} + \sigma^2\right)^{-1}\left(r_{i,j}^{DL}\right)^{-\alpha}}\right] \tag{5}$$

where $r_{i,j}^{DL} = \left\|u_{i,j}^{DL} - b_j\right\|$ denotes the distance from the nearest interfering BS to the typical downlink user, I_{agg} denotes the aggregated interference from downlink BSs transmissions. More specifically, I_{agg} is given in the following:

$$\begin{aligned} I_{agg}^{DL} = \mathbb{E}\left[\sum_{b_k \in \Phi_B / b_j} I_{b_k}\right] \overset{(a)}{=} \frac{\mathbb{E}\left[P_j^{DL}\right]}{N_j^{DL}} \lambda_b \int\limits_{x \in \mathbb{R}^2} \|x - b_k\|^{-\alpha} dx \\ = 2\frac{\mathbb{E}\left[P_j^{DL}\right]}{N_j^{DL}} \lambda_b \pi \int_{r_{i,j}^{DL}}^{\infty} r^{-\alpha} r dr = \frac{2\mathbb{E}\left[P_j^{DL}\right]\lambda_b \pi \left(r_{i,j}^{DL}\right)^{2-\alpha}}{N_j^{DL}(\alpha - 2)} \end{aligned} \tag{6}$$

where $I_{b_k}=\frac{\mathbb{E}\left[P_j^{DL}\right]}{N_j^{DL}}g_k^{DL}\left\|u_{i,j}^{DL}-b_k\right\|^{-\alpha}$, (a) follows Campbells Theorem [8] of PPP and $\mathbb{E}\left[g_k^{DL}\right]=1$. Therefore, the expectation of $P_{i,j}^{DL}$ is obtained using the probability density function (PDF) of $r_{i,j}^{DL}$ (i.e., $f_{r_{i,j}^{DL}}(r)=2\pi\lambda_b re^{-\pi\lambda_b r^2}$), as follows:

$$\begin{aligned}\mathbb{E}\left[P_{i,j}^{DL}\right]&=\int_0^\infty \mathbb{E}\left[P_{i,j}^{DL}\left|r=\left\|u_{i,j}^{DL}-b_j\right\|\right.\right]f_{r_{i,j}^{DL}}(r)\,dr\\ &=\mathbb{E}\left[\left(2^{R_{i,j}^{DL}/B_{i,j}^{DL}}-1\right)\left(\frac{2\mathbb{E}\left[P_j^{DL}\right]}{N_j^{DL}(\alpha-2)}+\frac{\sigma^2\Gamma\left(\frac{\alpha+2}{\alpha}\right)}{(\lambda_b\pi)^{\frac{\alpha}{2}}}\right)\right]\end{aligned}\tag{7}$$

where $\Gamma(x)=\int_0^\infty t^{x-1}e^{-t}dt$ is standard gamma function.

Note that the average number of cellular downlink users per BS is given by $\mathbb{E}\left[N_j^{DL}\right]=\lambda_u^{DL}/\lambda_b$, and we have averaged the allocated resources for each downlink user on $\mathbb{E}\left[B_{i,j}^{DL}\right]=B^{DL}/\mathbb{E}\left[N_j^{DL}\right]=B^{DL}\lambda_b/\lambda_u^{DL}$. To simplify the analysis, we constrain the rate requirement of users in cellular downlink to be equally averaged over R_u^{DL} (i.e., $\forall R_{i,j}^{DL}=R_u^{DL}$). Moreover, the average aggregated transmit power of per BS can be obtained by calculating the sum of per user's power (i.e., $\mathbb{E}\left[P_j^{DL}\right]=\mathbb{E}\left[N_j^{DL}\right]\mathbb{E}\left[P_{i,j}^{DL}\right]$) and is given by

$$\mathbb{E}\left[P_j^{DL}\right]=\frac{\lambda_b^{-\frac{\alpha+2}{2}}\sigma^2(\alpha-2)\lambda_u^{DL}\Gamma\left(\frac{\alpha+2}{\alpha}\right)}{\pi^{\frac{\alpha}{2}}\left[(\alpha-2)\left(2^{\frac{R_u^{DL}\lambda_u^{DL}}{B^{DL}\lambda_b}}-1\right)^{-1}-2\right]}.\tag{8}$$

B. Harvested Energy with Isotropic Mode

We leverage the Laplace transform inversion method to specifically derive the target function which is the PDF of harvested energy at an uplink user, as follows:

$$\mathcal{L}_{P_H}(s)=\mathbb{E}\left[\exp(-sP_H)\right]=\int_0^\infty e^{-st}f_{P_H}(t)\,dt\tag{9}$$

$$f_{P_H}(t)=\mathcal{L}^{-1}\left\{\mathcal{L}_{P_H}(s)\right\}\tag{10}$$

where $\mathcal{L}_{P_H}(s)$ denotes the Laplace transform of P_H defined in (3), $f_{P_H}(t)$ is the PDF of P_H and $\mathcal{L}^{-1}(\cdot)$ represents the inverse Laplace transform.

More specifically, recall that an uplink user can harvest energy from both downlink transmissions of BSs and microwave radiation of PBs, and thus $\mathcal{L}_{P_H}(s)$ can be expressed as

$$\mathcal{L}_{P_H}(s)=\mathcal{L}_{P_H^{BS}}(s)\,\mathcal{L}_{P_H^{PS}}(s)\tag{11}$$

where $\mathcal{L}_{P_H^{BS}}$ and $\mathcal{L}_{P_H^{PS}}$ denote the Laplace transform of P_H^{BS} and P_H^{PS} defined in (3), respectively.

Furthermore, according to the properties of PPP, $\mathcal{L}_{P_H^{BS}}$ can be given as follows:

$$\begin{aligned}\mathcal{L}_{P_H^{BS}}(s)&=\mathbb{E}\left[\mathrm{e}^{-sP_H^{BS}}\right]\overset{(a)}{=}\mathbb{E}_{\Phi_{B,g}}\left[\mathrm{e}^{-s\eta\sum\limits_{b_j\in\Phi_B}P_b^{agg}g_j^{DL}{x_j}^{-\alpha}}\right]\\ &=\exp\left[-\frac{2\lambda_b\left(\eta P_b^{agg}\right)^{\frac{2}{\alpha}}\pi^2}{\alpha\sin\left(\frac{2\pi}{\alpha}\right)}s^{\frac{2}{\alpha}}\right]\overset{(b)}{=}e^{-\xi_1 s^{\frac{2}{\alpha}}}\end{aligned}\tag{12}$$

where, in (a), $P_b^{agg}=\mathbb{E}\left[P_j^{DL}\right]$ is given in (8) and g_j^{DL} represents the fast-fading factor ($g_j^{DL}\sim$ exp(1)), and (b) follows from $\xi_1=\frac{2\lambda_b\left(\eta P_b^{agg}\right)^{\frac{2}{\alpha}}\pi^2}{\alpha\sin\left(\frac{2\pi}{\alpha}\right)}$.

In addition, $\mathcal{L}_{P_H^{PS}}$ and $\mathcal{L}_{P_H}(s)$ is obtained as follows:

$$\mathcal{L}_{P_H^{PB}}(s)=e^{-\xi_2 s^{\frac{2}{\beta}}},\mathcal{L}_{P_H}(s)=e^{-\left(\xi_1 s^{2/\alpha}+\xi_2 s^{2/\beta}\right)}\tag{13}$$

where $\xi_2=\frac{2\lambda_p(\eta P_h)^{\frac{2}{\beta}}\pi^2}{\beta\sin\left(\frac{2\pi}{\beta}\right)}$.

The cumulative distribution function (CDF) of P_H is

$$\begin{aligned}F_{P_H}(x)&=\mathbb{P}(P_H\le x)=\int_0^x f_{P_H}(t)dt\\ &=\int_0^x\frac{1}{2\pi i}\lim_{T\to\infty}\int_{c-iT}^{c+iT}e^{st}\mathcal{L}_{P_H}(s)\,dsdt\\ &=\frac{1}{2\pi i}\lim_{T\to\infty}\int_{c-iT}^{c+iT}\left(\frac{e^{sx}-1}{s}\right)e^{-\left(\xi_1 s^{2/\alpha}+\xi_2 s^{2/\beta}\right)}ds\\ &=\frac{1}{2\pi i}\lim_{T\to\infty}\int_{c-iT}^{c+iT}\frac{e^{sx-\left(\xi_1 s^{2/\alpha}+\xi_2 s^{2/\beta}\right)}}{s}ds.\end{aligned}\tag{14}$$

Considering equation (14) has a branch point at the origin, we use the Laplace inversion method to calculate the integral:

$$\begin{aligned}F_{P_H}(x)=&-\lim_{\substack{R\to\infty\\ r\to0}}\frac{1}{2\pi i}\left\{\int_\pi^{-\pi}ie^{re^{i\theta}x-\xi_1 r^{\frac{2}{\alpha}}e^{\frac{2i\theta}{\alpha}}}\right.\\ &\cdot e^{-\xi_2 r^{\frac{2}{\beta}}e^{\frac{2i\theta}{\beta}}}d\theta+\int_R^r e^{ue^{i\pi}x-\xi_1 u^{\frac{2}{\alpha}}e^{\frac{2i\pi}{\alpha}}-\xi_2 u^{\frac{2}{\beta}}e^{\frac{2i\pi}{\beta}}}\frac{du}{u}\\ &\left.+\int_r^R e^{ue^{-i\pi}x-\xi_1 u^{\frac{2}{\alpha}}e^{\frac{-2i\pi}{\alpha}}-\xi_2 u^{\frac{2}{\beta}}e^{\frac{-2i\pi}{\beta}}}\frac{du}{u}\right\}\\ =&1-\lim_{\substack{R\to\infty\\ r\to0}}\int_R^r\frac{e^{-ux}}{2\pi i}\left[e^{-\xi_1 u^{\frac{2}{\alpha}}[\theta_2+i\theta_1]-\xi_2 u^{\frac{2}{\beta}}[\phi_2+i\phi_1]}\right.\\ &\left.-e^{-\xi_1 u^{\frac{2}{\alpha}}[\theta_2-i\theta_1]-\xi_2 u^{\frac{2}{\beta}}[\phi_2-i\phi_1]}\right]\frac{du}{u}\\ =&1-\int_0^\infty\frac{\sin\left(\xi_1 u^{\frac{2}{\alpha}}\theta_1+\xi_2 u^{\frac{2}{\beta}}\phi_1\right)}{\pi\exp\left(ux+\xi_1 u^{\frac{2}{\alpha}}\theta_2+\xi_2 u^{\frac{2}{\beta}}\phi_2\right)}\frac{du}{u}\end{aligned}\tag{15}$$

where $\theta_1=\sin\left(\frac{2\pi}{\alpha}\right)$, $\theta_2=\cos\left(\frac{2\pi}{\alpha}\right)$, $\phi_1=\sin\left(\frac{2\pi}{\beta}\right)$, and $\phi_2=\cos\left(\frac{2\pi}{\beta}\right)$.

The PDF of P_H can be derived from (15) as

$$\begin{aligned}f_{P_H}(x)=&\frac{\partial F_{P_H}(x)}{\partial x}=\frac{1}{\pi}\int_0^\infty e^{-ux-\xi_1 u^{\frac{2}{\alpha}}\theta_2-\xi_2 u^{\frac{2}{\beta}}\phi_2}\\ &\cdot\sin\left(\xi_1 u^{\frac{2}{\alpha}}\theta_1+\xi_2 u^{\frac{2}{\beta}}\phi_1\right)du.\end{aligned}\tag{16}$$

Although $f_{P_H}(x)$ has one integration, it can be directly obtained by numerical calculations.

C. Harvested Energy with Directed Mode

Then we consider the directed mode of PBs (i.e., the uplink user can be charged by its nearest PB), and the corresponding expression for the distribution of P_H is derived in the sequel. Considering the practical average aggregated transmit power of

BS given in (8), we rewrite the instantaneous received power defined in (4) under the directed mode of PBs as follows:

$$\begin{aligned} P_H &= P_H^{PB} + P_H^{BS} = \eta G_m P_h g_j^h \|p_j - u_0\|^{-\beta} \\ &+\eta \sum_{b_j \in \Phi_B} P_b^{agg} g_j^{DL} \|b_j - u_0\|^{-\alpha} \end{aligned} \tag{17}$$

where $P_b^{agg} = \mathbb{E}\left[P_j^{DL}\right]$ is obtained in (8).

The CDF of P_H^{PB} can be derived as follows:

$$\begin{aligned} F_{P_H^{PB}}(y) &\overset{(a)}{=} \mathbb{P}\left(Y = \eta G_m P_h x^{-\beta} \le y\right) \\ &= \mathbb{P}\left[x \ge \left(\frac{\eta G_m P_h}{y}\right)^{\frac{1}{\beta}}\right] \overset{(b)}{=} e^{-\pi\lambda_p\left(\frac{\eta G_m P_h}{y}\right)^{\frac{2}{\beta}}} \end{aligned} \tag{18}$$

where $x = \|p_j - u_0\|$ in (a), and (b) follows from the distribution of $\|p_j - u_0\|$ (i.e., $f_{\|p_j - u_0\|}(r) = 2\pi\lambda_p r e^{-\pi\lambda_p r^2}$), since we assume that users are powered by the closest PB with beamforming. Taking the derivative of (18) with respect to y, we get the PDF of P_H^{PB} as

$$f_{P_H^{PB}}(y) = \frac{\partial F_{P_H^{PB}}(y)}{\partial y} = \frac{2\pi\lambda_p(\eta G_m P_h)^{\frac{2}{\beta}}}{\beta y^{\frac{2}{\beta}+1} \exp\left[\pi\lambda_p(\eta G_m P_h/y)^{\frac{2}{\beta}}\right]}. \tag{19}$$

Using Laplace transform inversion method, we can get the PDF of P_H^{BS} (i.e., $f_{P_H^{PB}}(y)$), which is similar to (16) as follows:

$$f_{P_H^{BS}}(x) = \frac{1}{\pi}\int_0^\infty e^{-ux-\xi_1 u^{\frac{2}{\alpha}}\theta_2} \sin\left(\xi_1 u^{\frac{2}{\alpha}}\theta_1\right) du. \tag{20}$$

The CDF of the instantaneous received power P_H is

$$\begin{aligned} F_{P_H}(z) &= \int_{0^+}^{z}\int_{0^+}^{z-y} f_{P_H^{BS}}(x) f_{P_H^{PB}}(y)\, dxdy \\ &= \frac{2\lambda_p(\eta G_m P_h)^{\frac{2}{\beta}}}{\beta} \int_{0^+}^{z}\int_{0^+}^{z-y} y^{-\frac{2}{\beta}-1} e^{-\pi\lambda_p(\eta G_m P_h/y)^{\frac{2}{\beta}}} \\ &\cdot \int_0^\infty e^{-ux-\xi_1 u^{\frac{2}{\alpha}}\theta_2} \sin\left(\xi_1 u^{\frac{2}{\alpha}}\theta_1\right) dudxdy. \end{aligned} \tag{21}$$

The PDF of P_H can be derived as follows:

$$\begin{aligned} f_{P_H}(z) &= \frac{\partial F_{P_H}(z)}{\partial z} = \frac{2\rho}{\beta}\int_{0^+}^{z}\int_0^\infty e^{-\pi\rho y^{\frac{-2}{\beta}}} \\ &\cdot y^{-\frac{2}{\beta}-1} e^{-u(z-y)-\xi_1 u^{\frac{2}{\alpha}}\theta_2} \sin\left(\xi_1 u^{\frac{2}{\alpha}}\theta_1\right) dudy \end{aligned} \tag{22}$$

where $\rho = \lambda_p(\eta G_m P_h)^{\frac{2}{\beta}}$, $\xi_1 = \frac{2\lambda_b\left(\eta P_b^{agg}\right)^{\frac{2}{\alpha}}\pi^2}{\alpha\sin\left(\frac{2\pi}{\alpha}\right)}$. $f_{P_H}(z)$ involves a double integral which can be evaluated numerically.

In addition, the average transmit power of a cellular uplink user (i.e., the expectation of harvested energy P_H) is obtained as

$$P_T = \mathbb{E}[P_H] = \int_{0^+}^\infty x f_{P_H}(x) dx \tag{23}$$

where $f_{P_H}(x)$ is in (16) with isotropic mode or (22) with directed mode.

IV. ANALYSIS OF SUCCESSFUL TRANSMISSION PROBABILITY

In this section, we utilize the distribution of harvested energy P_H to obtain the successful transmission probability of an uplink user under the practical data rate requirement of cellular networks. It's worth noting that the outage for uplink users may occur due to either insufficient amount of harvested energy, or the desired transmit power exceeds the MATP at device.

The average transmit power of a typical uplink user $u_{i,j}^{UL}$ with the required data rate $R_{i,j}^{UL}$ can be obtained by a transformation of equation (1) conditioned on the service link distance $r_{i,j}^{UL} = \left\|u_{i,j}^{UL} - b_j\right\|$ as follows:

$$\mathbb{E}\left[P_{i,j}^{UL}\left|r_{i,j}^{UL}\right.\right] = \mathbb{E}\left[\left(2^{R_{i,j}^{UL}/B_{i,j}^{UL}} - 1\right)\cdot\frac{\left(I_{agg}^{UL}+\sigma^2\right)}{(r_{i,j}^{UL})^{-\alpha}}\right] \tag{24}$$

where I_{agg}^{UL} indicates the average aggregated interference, and we have utilized $\mathbb{E}\left[h_{i,j}^{UL}\right] = 1$. Furthermore, I_{agg}^{UL} can be obtained by applying Campbells Theorem, as follows:

$$I_{agg}^{UL} = \mathbb{E}\left[\sum_{u_n^{UL}\in\Phi_{I_{i,j}}^{UL}} \frac{P_n^T g_n^{UL}}{(d_n^{UL})^\alpha}\right] = \frac{2P_T\lambda_b\pi R^{2-\alpha}}{\alpha-2} \tag{25}$$

where $\mathbb{E}\left[g_n^{UL}\right] = 1$, for arbitrary n, $P_n^T = P_T = \mathbb{E}[P_H]$, and P_T is given in (23). Note that the intensity of interfering users is equal to the density of BSs, since the transmission links are orthogonal within a cell. $R = \frac{1}{\sqrt{\pi\lambda_b}}$ represents the approximate radius of cell area, and it can be regarded as the distance of the nearest interfering device for the receiver BS.

The distribution of the desired transmit power for a typical uplink user $u_{i,j}^{UL}$ with the required data rate $R_{i,j}^{UL}$ is

$$\begin{aligned} F_{P_{i,j}^{UL}}(x) &= \mathbb{P}\left(\mathbb{E}\left[P_{i,j}^{UL}\left|\left\|u_{i,j}^{UL} - b_j\right\|\right.\right] \le x\right) \\ &= \mathbb{P}\left\{\mathbb{E}\left[\frac{\left(2^{R_{i,j}^{UL}/B_{i,j}^{UL}}-1\right)\left(2P_T(\lambda_b\pi)^{\frac{\alpha}{2}}+(\alpha-2)\sigma^2\right)}{\left\|u_{i,j}^{UL}-b_j\right\|^{-\alpha}(\alpha-2)}\right] \le x\right\} \\ &\overset{(a)}{=} \mathbb{P}\left\{r_{i,j}^{UL} \le \mathbb{E}\left[\left[\frac{x(\alpha-2)\left(2^{R_{i,j}^{UL}/B_{i,j}^{UL}}-1\right)^{-1}}{\left(2P_T(\lambda_b\pi)^{\frac{\alpha}{2}}+(\alpha-2)\sigma^2\right)}\right]^{\frac{1}{\alpha}}\right]\right\} \\ &\overset{(b)}{=} 1 - e^{-\pi\lambda_b(x\omega)^{\frac{2}{\alpha}}} \end{aligned} \tag{26}$$

where $\omega = \mathbb{E}\left[\frac{(\alpha-2)\left(2^{R_{i,j}^{UL}/B_{i,j}^{UL}}-1\right)^{-1}}{2P_T(\lambda_b\pi)^{\frac{\alpha}{2}}+(\alpha-2)\sigma^2}\right]$, $r_{i,j}^{UL} = \left\|u_{i,j}^{UL} - b_j\right\|$ in (a), and (b) follows from the PDF of PPP (i.e., $f_{r_{i,j}^{UL}}(r) = 2\pi\lambda_b r\exp\left(-\pi\lambda_b r^2\right)$, $r > 0$, and thus $\mathbb{P}\left(r_{i,j}^{UL} \le r\right) = F_{r_{i,j}^{UL}}(r) = \int_0^r f_{r_{i,j}^{UL}}(r)\, dr = 1 - e^{-\pi\lambda_b r^2}$).

Therefore, the PDF of the desired transmit power $P_{i,j}^{UL}$ of the uplink user $u_{i,j}^{UL}$ is given by

$$f_{P_{i,j}^{UL}}(x) = \frac{\partial F_{P_{i,j}^{UL}}(x)}{\partial x} = \pi\lambda_b\frac{2}{\alpha}\omega^{\frac{2}{\alpha}}x^{\frac{2-\alpha}{\alpha}}e^{-\pi\lambda_b(x\omega)^{\frac{2}{\alpha}}}. \tag{27}$$

As mobile device has the MATP P_u, the PDF of $P_{i,j}^{UL}$ with limited transmit power (i.e., $0 \leq P_{i,j}^{UL} \leq P_u$) is

$$\begin{aligned} f_{P_{i,j}^{UL}}(x)\,|_{P_u} &= \frac{f_{P_{i,j}^{UL}}(x)}{\int_0^{P_u} f_{P_{i,j}^{UL}}(y)\,dy} \\ &= \frac{2\pi\lambda_b \omega^{\frac{2}{\alpha}} x^{\frac{2-\alpha}{\alpha}} e^{-\pi\lambda_b (x\omega)^{\frac{2}{\alpha}}}}{\alpha\left(1 - e^{-\pi\lambda_b (P_u\omega)^{\frac{2}{\alpha}}}\right)} \end{aligned} \tag{28}$$

and thus the distribution of the desired transmit power at an arbitrary mobile device with MATP P_u is obtained as

$$F_{P_{i,j}^{UL}}(x)\,|_{P_u} = \frac{1 - e^{-\pi\lambda_b (x\omega)^{\frac{2}{\alpha}}}}{1 - e^{-\pi\lambda_b (P_u\omega)^{\frac{2}{\alpha}}}}. \tag{29}$$

In addition, the average desired transmit power of a user is

$$\begin{aligned} \mathbb{E}\left[P_{i,j}^{UL}\right] &= \int_0^{P_u} x f_{P_{i,j}^{UL}}(x)\,|_{P_u}\, dx \\ &= \frac{\gamma\left(\frac{2+\alpha}{\alpha}, -\pi\lambda_b (P_u\omega)^{\frac{2}{\alpha}}\right)}{\left(1 - e^{-\pi\lambda_b (P_u\omega)^{\frac{2}{\alpha}}}\right)(\pi\lambda_b)^{\frac{\alpha}{2}}\omega} \end{aligned} \tag{30}$$

where $\gamma(x, z)$ is the incomplete gamma function as the following $\gamma(z, x) = \int_0^x t^{z-1} e^{-t} dt$.

We define the maximum power outage probability $\mathcal{O}_P$, which occurs because the desired transmit power at a mobile device exceeds the MATP P_u, as follows:

$$\begin{aligned} \mathcal{O}_P &= \mathbb{P}\left(\mathbb{E}\left[P_{i,j}^{UL}\,\middle|\,\left\|u_{i,j}^{UL} - b_j\right\|\right] > P_u\right) \\ &= \exp\left[\frac{-\lambda_b\pi\left(2^{\left(R_u^{UL}\lambda_u^{UL}\right)/\left(B^{UL}\lambda_b\right)} - 1\right)^{\frac{-2}{\alpha}}}{\left(\frac{2P_T(\lambda_b\pi)^{\frac{\alpha}{2}}}{P_u(\alpha-2)} + \frac{\sigma^2}{P_u}\right)^{\frac{2}{\alpha}}}\right] \end{aligned} \tag{31}$$

where (31) can be obtained from (26) with minor modifications. Similar to the cellular downlink, $\mathbb{E}\left[N_j^{UL}\right] = \lambda_u^{UL}/\lambda_b$, $\forall R_{i,j}^{UL} = R_u^{UL}$ and $\mathbb{E}\left[B_{i,j}^{UL}\right] = B^{UL}\lambda_b/\lambda_u^{UL}$.

Though different modes in operation, the expressions of the maximum power outage probability $\mathcal{O}_P$ are the same expect that P_T is different.

Moreover, we define the energy harvesting outage probability $\mathcal{O}_H$, which can happen due to insufficient capture power at a mobile device.

1) Isotropic Mode: Combining (16) and (29), we can obtain $\mathcal{O}_H$ in isotropic mode as follows:

$$\begin{aligned} \mathcal{O}_H &= \mathbb{P}\left(P_H < \mathbb{E}\left[P_{i,j}^{UL}\right]\right) \\ &= 1 - \int_{0+}^{\infty} F_{P_{i,j}^{UL}}(x)\,|_{P_u}\, f_{P_H}(x)dx \\ &= 1 - \frac{\pi^{-1}}{1 - e^{-\pi\lambda_b (P_u\omega)^{\frac{2}{\alpha}}}} \int_{0+}^{\infty}\int_0^{\infty}\left(1 - e^{-\pi\lambda_b (x\omega)^{\frac{2}{\alpha}}}\right) \\ &\cdot e^{-ux-\xi_1 u^{\frac{2}{\alpha}}\theta_2 - \xi_2 u^{\frac{2}{\beta}}\phi_2} \sin\left(\xi_1 u^{\frac{2}{\alpha}}\theta_1 + \xi_2 u^{\frac{2}{\beta}}\phi_1\right) dudx. \end{aligned} \tag{32}$$

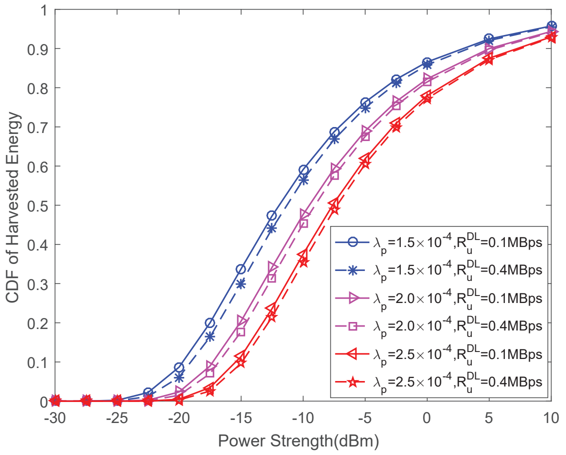

Fig. 2. Distribution of harvested energy with isotropic mode of PBs under varying densities of PBs (λ_p) and downlink desired data rate (R_u^{DL}).

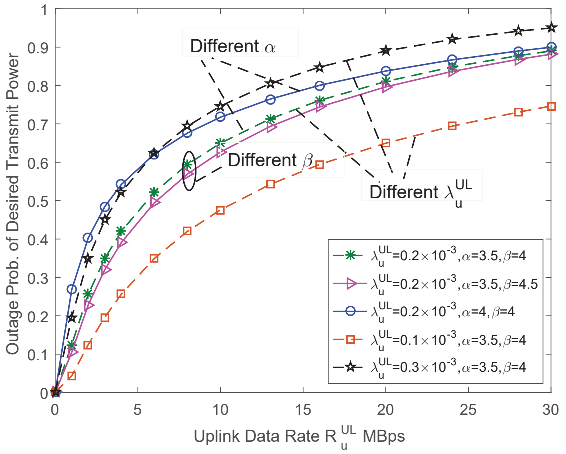

Fig. 3. Probability that uplink desired transmit power ($P_{i,j}^{UL}$) exceeds MATP at device (P_u) with isotropic mode of PBs.

2) Directed Mode: Combining (22) and (29), we can obtain $\mathcal{O}_H$ in directed mode as follows:

$$\begin{aligned} \mathcal{O}_H &= \mathbb{P}\left(P_H < \mathbb{E}\left[P_{i,j}^{UL}\right]\right) \\ &= 1 - \int_{0+}^{\infty} F_{P_{i,j}^{UL}}(z)\,|_{P_u}\, f_{P_H}(z)dz \\ &= 1 - \frac{2\beta^{-1}\rho}{1 - e^{-\pi\lambda_b (P_u\omega)^{\frac{2}{\alpha}}}} \int_{0+}^{\infty}\int_{0+}^{z}\int_0^{\infty}\left(1 - e^{-\pi\lambda_b (z\omega)^{\frac{2}{\alpha}}}\right) \\ &\cdot y^{\frac{-2-\beta}{\beta}} e^{-\pi\rho y^{\frac{-2}{\beta}}} \sin\left(\xi_1 u^{\frac{2}{\alpha}}\theta_1\right) e^{-u(z-y)-\xi_1 u^{\frac{2}{\alpha}}\theta_2} dudydz. \end{aligned} \tag{33}$$

Furthermore, combining (31) and (32) or (33), we can obtain the successful transmission probability that an uplink user can transmit while satisfying its QoS requirement as follows:

$$\mathcal{P}_{suc} = \left(1 - \mathcal{O}_P\right)\left(1 - \mathcal{O}_H\right). \tag{34}$$

V. NUMERICAL SIMULATIONS AND DISCUSSIONS

In this section, we evaluate the performance of the proposed WPCN model with randomly deployed PBs, by numerical simulations with diverse parameter settings given practical

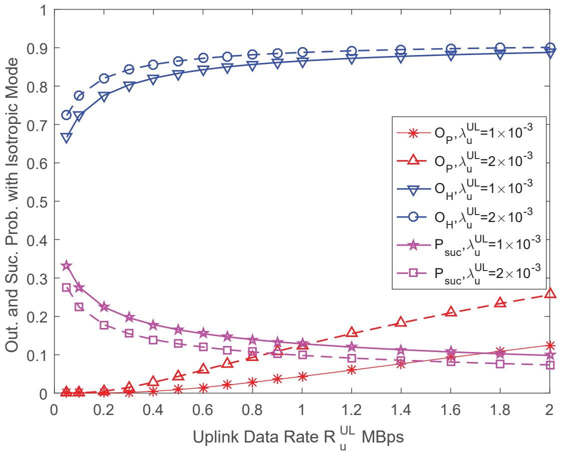

Fig. 4. Maximum power outage Prob. ($\mathcal{O}_P$), energy harvesting outage Prob. ($\mathcal{O}_H$) and successful transmission Prob. ($\mathcal{P}_{suc}$) regarding to the uplink desired data rate R_u^{UL}.

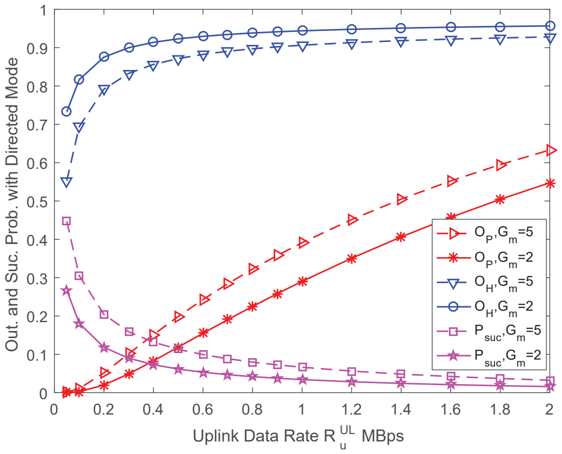

Fig. 5. $\mathcal{O}_P$, $\mathcal{O}_H$ and $\mathcal{P}_{suc}$ regarding to the uplink desired data rate R_u^{UL} with directed mode of PBs.

network scenarios. We generally select $B^{UL} = 30MHz$, $B^{DL} = 30MHz$, $\alpha = 3.5$, $\beta = 4$, $\lambda_b = 5 \times 10^{-5} BSs/m^2$, $P_b = 40W$, $\lambda_p = 2 \times 10^{-4} PBs/m^2$, $P_h = 100W$, $\eta = 1$, $\lambda_u^{UL} = 2\times10^{-3} users/m^2$, $P_u = 100mW$, $R_u^{UL} = 0.1Mbps$, $R_u^{DL} = 0.4Mbps$, $\sigma^2 = -80dBm$, unless specified otherwise.

Fig. 2 illustrates the distribution of harvested energy with isotropic mode of PBs. It can be observed that increasing the intensity of PBs or downlink users data rate requirement leads to higher harvested energy at uplink users.

Fig. 3 shows the maximum power outage probability ($\mathcal{O}_P$) with isotropic mode of PBs, regarding to the uplink required data rate (R_u^{UL}) under various uplink user intensities (λ_u^{UL}) and path-loss factors(α and β). It can be observed that the outage probability increases with R_u^{UL} because it is easier to achieve MATP at device with larger R_u^{UL}. In addition, with λ_u^{UL} and α increasing, the outage probability is also growing due to the fact that more uplink users and greater α consume more power. On the other hand, when the poss-loss factor β increases, the transmit power of the interfering users (average harvested energy) could be reduced, which attenuates the interference severely and thus decreases the outage probability conversely.

Moreover, maximum power outage probability ($\mathcal{O}_P$), energy harvesting outage probability ($\mathcal{O}_H$) and successful transmission probability ($\mathcal{P}_{suc}$) are investigated in Fig. 4 and Fig. 5 for isotropic mode and directed mode of PBs, respectively. It is clear that in different modes, the trends of $\mathcal{O}_P$, $\mathcal{O}_H$ or $\mathcal{P}_{suc}$ are almost the same respectively. In different modes, $\mathcal{O}_P$ and $\mathcal{O}_H$ increase with R_u^{UL}, while $\mathcal{P}_{suc}$ decreases as R_u^{UL} grows. In the isotropic mode (Fig. 4), when λ_u^{UL} rises, $\mathcal{O}_P$ and $\mathcal{O}_H$ increase, and $\mathcal{P}_{suc}$ declines. In the directed mode (Fig. 5), $\mathcal{P}_{suc}$ rises with the efficiency of the beam main-lobe (G_m) because greater G_m causes lager harvested energy at mobile devises, and on the contrary $\mathcal{O}_P$ and $\mathcal{O}_H$ could decrease.

VI. CONCLUSION

In this paper, we evaluate the performance of the wireless powered cellular networks with randomly deployed PBs, while incorporating the practical data rate requirement for both uplink and downlink transmissions. The cellular networks and wireless-powered systems are modeled by leveraging the tools of stochastic geometry. More importantly, network performance indicators such as average transmit power, outage probability and successful transmission probability are derived under isotropic mode and directed mode of power beacons, respectively. Finally, numerical results demonstrate our theoretical analysis and express the effectiveness of the proposed model.

ACKNOWLEDGMENT

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